

Spin Hall Current and Spin-transfer Torque in Ferromagnetic Metal

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Abstract. We theoretically examine the spin-transfer torque in the presence of spin-orbit interaction (SOI) at impurities in a ferromagnetic metal on the basis of linear response theory. We obtained, in addition to the usual spin-transfer torque, a new contribution $\sim \mathbf{j}_{\text{SH}} \cdot \nabla \mathbf{n}$ in the first order in SOI, where \mathbf{j}_{SH} is the spin Hall current driven by an external electric field. This is a reaction to inverse spin Hall effect driven by spin motive force in a ferromagnet.

1. Introduction

Since the theoretical proposal by Slonczewski [1] and Berger [2] of the current-induced magnetization reversal in nanopillars and subsequent experimental realizations, the spin current has been recognized to be useful in manipulating magnetization in tiny magnets. An important concept is the spin-transfer torque that the spin current exerts on magnetization. For a smooth spin texture \mathbf{n} , it is expressed as $(\hbar/2e) \mathbf{j}_s \cdot \nabla \mathbf{n}$ [3, 4]. Here $e > 0$ is the elementary charge, $\mathbf{j}_s = \sigma_s \mathbf{E}$ is the spin-current density driven by an applied electric field \mathbf{E} , and $\sigma_s = \sigma_{\uparrow} - \sigma_{\downarrow}$ is the “spin conductivity” with σ_{\uparrow} (σ_{\downarrow}) being the diagonal conductivity for majority-spin (minority-spin) electrons.

Recently, we theoretically studied the spin Hall current [5] in a ferromagnetic conductor in the presence of spin texture \mathbf{n} and SOI at impurities [6]. It was shown that an electric field \mathbf{E} also induces a spin Hall current, $\mathbf{j}_{\text{SH}} = \sigma_{\text{SH}} \mathbf{n} \times \mathbf{E}$, where $\sigma_{\text{SH}} = \sigma_{\text{H}\uparrow} + \sigma_{\text{H}\downarrow}$ is the spin Hall conductivity with $\sigma_{\text{H}\uparrow}$ ($\sigma_{\text{H}\downarrow}$) being Hall conductivity for majority-spin (minority-spin) electrons [7]. The total spin current \mathcal{J}_S is thus given by

$$\mathcal{J}_S = \mathbf{j}_s + \mathbf{j}_{\text{SH}} = \sigma_s \mathbf{E} + \sigma_{\text{SH}} \mathbf{n} \times \mathbf{E}. \quad (1)$$

Naturally, this spin Hall current (\mathbf{j}_{SH}) is expected to contribute to the spin-transfer torque as

$$\mathbf{t}_{\text{el}}^{\text{H}} = \frac{\hbar}{2e} (\mathbf{j}_s + \mathbf{j}_{\text{SH}}) \cdot \nabla \mathbf{n}. \quad (2)$$

The spin Hall conductivity σ_{SH} in Eq. (1) is given by the correlation function between spin current and charge current, and its reciprocal effect (in the sense of Onsager) described by the same function is the inverse spin Hall effect which is driven by spin motive force (SMF) [8, 9, 10, 11, 12, 13, 14] or a spin-dependent effective electric field \mathbf{E}_s [12, 13], and induces a charge current

$$\mathcal{J} = \sigma_s \mathbf{E}_s + \sigma_{\text{SH}} \mathbf{n} \times \mathbf{E}_s. \quad (3)$$

Such a spin-dependent field \mathbf{E}_s is known to arise from the dynamics of textured magnetization, and is given by $E_{s,i} = (\hbar/2e) \mathbf{n} \cdot (\partial_i \mathbf{n} \times \dot{\mathbf{n}})$. The first term of Eq.(3) is actually the reciprocal effect of the ordinary spin-transfer effect (first term of Eq. (2)) [12].

The purpose of this paper is to derive the second term of Eq. (2) microscopically and clarify the relation to the corresponding SMF (second term of Eq. (3)), both of which arise in the presence of SOI.

2. Model and Calculation

We consider the s - d model with conducting s electrons and localized d spins, \mathbf{n} , which are coupled via the s - d exchange interaction, $-M\mathbf{n} \cdot \boldsymbol{\sigma}$. The s electrons are subjected to impurity potential $V_{\text{imp}}(\mathbf{r}) = u \sum_i \delta(\mathbf{r} - \mathbf{R}_i)$, where u and \mathbf{R}_i are the strength and position of the impurity, as well as SOI, $\sim i\lambda_{\text{so}} \boldsymbol{\sigma} \cdot (\nabla \times V_{\text{imp}})$, at impurities [6]. In order to treat electrons in a spin texture, we perform a local transformation in electron spin space and take the spin quantization axis to be the d -spin direction \mathbf{n} at each point of space and time [15, 16, 17]. The Lagrangian in the rotated frame is given by [18]

$$L_{\text{el}} = \int d\mathbf{r} a^\dagger(x) \left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 + \varepsilon_F + M\sigma^z - V_{\text{imp}}(\mathbf{r}) \right] a(x), \quad (4)$$

$$\tilde{H}_{\text{so}} = \lambda_{\text{so}} \frac{m}{\hbar} \varepsilon_{ij\alpha} \int d\mathbf{r} (\partial_i V_{\text{imp}}(\mathbf{r})) \mathcal{R}^{\alpha\beta}(x) \tilde{j}_j^\beta(x), \quad (5)$$

$$H_{\text{e-A}} = \int d\mathbf{r} \tilde{j}_i^\alpha(x) A_i^\alpha(x). \quad (6)$$

Here $a^\dagger(x) = (a^\dagger_\uparrow(x), a^\dagger_\downarrow(x))$ is the electron creation operator at $x = (\mathbf{r}, t)$ in the rotated frame, σ^α 's are Pauli matrices, ε_F is the Fermi energy, λ_{so} is the strength of SOI, $\varepsilon_{ij\alpha}$ is the complete antisymmetric tensor with $\varepsilon_{xyz} = 1$, $\mathcal{R}^{\alpha\beta} = 2m^\alpha m^\beta - \delta^{\alpha\beta}$ is a 3×3 orthogonal matrix (with $\mathbf{m} = (\sin(\theta/2) \cos \phi, \sin(\theta/2) \sin \phi, \cos(\theta/2))$ for $\mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$), $\tilde{j}_i^\alpha = (\hbar/2mi)(a^\dagger \sigma^\alpha \partial_i a - \partial_i a^\dagger \sigma^\alpha a)$ is the spin-current density operator and $A_i^\alpha = (\mathbf{m} \times \partial_i \mathbf{m})^\alpha$ is the SU(2) gauge field. Repeated indices imply summation over $i, j, \alpha = x, y, z$.

In the s - d model, the spin torque is generally given by [19]

$$\mathbf{t}_{\text{el}}(x) = M\mathbf{n}(x) \times \mathcal{R}(x) \langle \tilde{\boldsymbol{\sigma}}(x) \rangle_{\text{ne}}, \quad (7)$$

where $\langle \tilde{\boldsymbol{\sigma}}(x) \rangle_{\text{ne}} = \langle a^\dagger(x) \boldsymbol{\sigma} a(x) \rangle_{\text{ne}}$ is the spin polarization of s electrons evaluated in an appropriate non-equilibrium state. For current-induced torques, it is calculated as a linear response to \mathbf{E} as

$$\langle \tilde{\boldsymbol{\sigma}}^\mu(\mathbf{q}) \rangle_{\text{ne}} = \lim_{\omega \rightarrow 0} \frac{\chi_i^\mu(\mathbf{q}, \omega + i0) - \chi_i^\mu(\mathbf{q}, 0)}{i\omega} E_i, \quad (8)$$

where $\chi_i^\mu(\mathbf{q}, \omega)$ is the current-spin correlation function obtained, for example, through an analytic continuation from the Matsubara representation

$$\chi_i^\mu(\mathbf{q}, i\omega_\lambda) = \int_0^{1/T} d\tau e^{i\omega_\lambda \tau} \langle T_\tau \tilde{\boldsymbol{\sigma}}^\mu(\mathbf{q}, \tau) J_i \rangle. \quad (9)$$

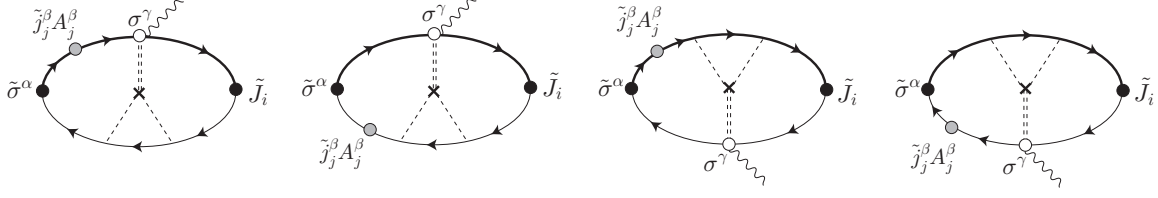


Figure 1. Feynman diagrams for the coefficients, χ_i^μ , of the spin torque induced by spin Hall current due to skew scattering. The thick (thin) solid line represents an electron line carrying Matsubara frequency $i\varepsilon_n + i\omega_\lambda$ ($i\varepsilon_n$). The dotted line (double dotted line with an open circle) represents potential (spin-orbit) scattering V_{imp} (\tilde{H}_{so}) by impurities. The gray circle represents the interaction with the SU(2) gauge field. The wavy line represents the rotation matrix $\mathcal{R}^{\alpha\beta}$.

Here T is the temperature, which will be eventually set to zero, $\omega_\lambda = 2\pi\lambda T$ with λ being an integer, and J_i is the current operator.

In this paper, we focus on the skew-scattering process (in the terminology of anomalous Hall effect [20]) and neglect the side-jump process. This corresponds to taking as the current operator $J_i \simeq \tilde{J}_i \equiv -e \sum_{\mathbf{k}} (\hbar \mathbf{k}_i / m) a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ neglecting the anomalous velocity term due to SOI. In the lowest order in λ_{so} , the first contribution to χ_i^μ comes from the third-order impurity scattering [21] and is first order in both \tilde{H}_{so} and $H_{\text{e-A}}$. The diagrammatic expressions are shown in Fig. 1. After some calculations, we obtain

$$\begin{aligned} \chi_i^\mu(\mathbf{q}, i\omega_\lambda) = & -ien_i u^3 \lambda_{\text{so}} \frac{4}{9\hbar} \varepsilon_{ij\lambda} \sum_{\mathbf{q}'} n_{\mathbf{q}-\mathbf{q}'}^\lambda A_j^\nu(\mathbf{q}') \\ & \times T \sum_n \sum_\sigma \sigma (\delta_\perp^{\mu\nu} + i\sigma \varepsilon_\perp^{\mu\nu}) \{ J_3^\sigma J_2^\sigma I_1^\sigma - (J_3^\sigma J_2^\sigma I_1^\sigma)^* \}, \end{aligned} \quad (10)$$

where $n_{\mathbf{q}}^\alpha$ is the Fourier component of the spin texture $n^\alpha(\mathbf{r})$ (note that $n^\alpha = \mathcal{R}^{\alpha z}$), $\sigma = \uparrow, \downarrow$ corresponds, respectively, to $\sigma = +1, -1$ in the formula (and to $\bar{\sigma} = \downarrow, \uparrow$ or $-1, +1$), $\delta_\perp^{\mu\nu} = \delta^{\mu\nu} - \delta^{\mu z} \delta^{\nu z}$, $\varepsilon_\perp^{\mu\nu} = -\varepsilon_\perp^{\nu\mu}$ with $\varepsilon_\perp^{xy} = 1$. We have defined $J_3^\sigma = \sum_{\mathbf{k}} G_{\mathbf{k}\sigma}^+ G_{\mathbf{k}\sigma}^+ G_{\mathbf{k}\sigma}$, $J_2^\sigma = \sum_{\mathbf{k}} G_{\mathbf{k}\sigma}^+ G_{\mathbf{k}\sigma}$, $I_1^\sigma = \sum_{\mathbf{k}} G_{\mathbf{k}\sigma}^+$, where $G_{\mathbf{k}\sigma}(z) = (z - \varepsilon_{\mathbf{k}} + \varepsilon_{\text{F}\sigma} + i\gamma_\sigma \text{sgn}(\text{Im}z))^{-1}$ and $G_{\mathbf{k}\sigma}^+ \equiv G_{\mathbf{k}\sigma}(i\varepsilon_n + i\omega_\lambda)$ are the impurity-averaged thermal Green's functions, with $\varepsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / 2m$, $\varepsilon_{\text{F}\sigma} = \varepsilon_{\text{F}} + \sigma M$, $\gamma_\sigma = \hbar / 2\tau_\sigma = \pi n_i u^2 \nu_\sigma$ and $\nu_\sigma = (m / 2\pi^2 \hbar^2) \sqrt{2m\varepsilon_{\text{F}\sigma}} / \hbar$.

After analytic continuation, $i\omega_\lambda \rightarrow \omega + i0$, we evaluate as $J_3^\sigma = (\sigma / 2M) 3\pi n_\sigma^{\text{el}} \tau_\sigma$, $J_2^\sigma = 3\pi n_\sigma^{\text{el}}$ and $I_1^\sigma = -i\pi \nu_\sigma$ ($n_\sigma^{\text{el}} = (2/3) \nu_\sigma \varepsilon_{\text{F}\sigma}$) in the lowest order of $\gamma_\sigma / \varepsilon_{\text{F}\sigma}$ and γ_σ / M . Then we obtain

$$\chi_i^\mu(\mathbf{q}, \omega + i0) - \chi_i^\mu(\mathbf{q}, 0) = i\omega \frac{1}{M} \frac{\hbar}{e} \sigma_{\text{SH}} \varepsilon_{ij\lambda} \sum_{\mathbf{q}'} n_{\mathbf{q}-\mathbf{q}'}^\lambda \delta_\perp^{\mu\nu} A_j^\nu(\mathbf{q}'), \quad (11)$$

where $\sigma_{\text{SH}} = \sigma_{\uparrow}^{\text{skew}} + \sigma_{\downarrow}^{\text{skew}}$ is the spin Hall conductivity with

$$\sigma_{\uparrow(\downarrow)}^{\text{skew}} = \lambda_{\text{so}} u \frac{2\pi e^2}{\hbar} (n_{\uparrow(\downarrow)}^{\text{el}})^2 \tau_{\uparrow(\downarrow)} \quad (12)$$

Thus the spin polarization is given by

$$\langle \tilde{\sigma}^\mu(x) \rangle_{\text{ne}} = \frac{1}{M} \frac{\hbar}{e} j_{\text{SH},i}(x) \delta_\perp^{\mu\nu} A_j^\nu(x), \quad (13)$$

where $\mathbf{j}_{\text{SH}}(x) = \sigma_{\text{SH}} \mathbf{n}(x) \times \mathbf{E}$ is the spin Hall current density [6]. Substituting Eq. (13) into Eq. (7) and using the relation, $\mathcal{R}^{\gamma\mu} \delta_{\perp}^{\mu\nu} A_i^{\nu} = -(\mathbf{n} \times \partial_i \mathbf{n})^{\gamma} / 2$ [19], we obtain

$$\begin{aligned} t_{\text{el}}^{\text{H},\alpha}(x) &= M \varepsilon_{\alpha\beta\gamma} n^{\beta}(x) \mathcal{R}^{\gamma\mu}(x) \frac{1}{M} \frac{\hbar}{e} \mathbf{j}_{\text{SH}}(x) \delta_{\perp}^{\mu\nu} A_i^{\nu}(x) \\ &= \frac{\hbar}{2e} \mathbf{j}_{\text{SH}}(x) \cdot \nabla n^{\alpha}(x). \end{aligned} \quad (14)$$

This is the desired spin-transfer torque due to spin Hall current. Combining with the ordinary spin-transfer torque, $(\hbar/2e) \mathbf{j}_s \cdot \nabla \mathbf{n}$ [4], and Eq. (14), the total spin-transfer torque in a disordered ferromagnetic metal is given by Eq. (2) with Eq. (1) .

3. Summary and discussion

We have shown that the spin-transfer torque in the presence of SOI at impurities has a contribution from the spin Hall current, given by the second term of Eq.(2). The result is consistent with the picture that it is a reciprocal effect to the corresponding SMF (second term of Eq. (3)), since both are characterized by the same coefficient σ_{SH} .

We have considered only the skew-scattering process, but not the side-jump process. Calculation of the latter process seems quite complicated and less straightforward compared to the former, in that the anomalous current is absent at the left (spin) vertex in the diagram, as opposed to the case of anomalous Hall conductivity. The whole calculation including the side-jump process will be reported in a future publication [22].

Acknowledgments

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